

DARK MATTER HALOS: VELOCITY ANISOTROPY – DENSITY SLOPE RELATION

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ABSTRACT

Dark matter (DM) halos formed in CDM cosmologies seem to be characterized by a power law phase-space density profile. The density of the DM halos is often fitted by the NFW profile but a better fit is provided by the Sersic fitting formula. These relations are empirically derived from cosmological simulations of structure formation but have not yet been explained on a first principle basis. Here we solve the Jeans equation under the assumption of a spherical DM halo in dynamical equilibrium, that obeys a power law phase space density and either the NFW-like or the Sersic density profile. We then calculate the velocity anisotropy, $\beta(r)$, analytically. Our main result is that for the NFW-like profile the $\beta - \gamma$ relation is not a linear one (where γ is the logarithmic derivative of the density $\rho[r]$). The shape of $\beta(r)$ depends mostly on the ratio of the gravitational to kinetic energy within the NFW scale radius R_s . For the Sersic profile a linear $\beta - \gamma$ relation is recovered, and in particular for the Sersic index of $n \approx 6.0$ case the linear fit of Hansen & Moore is reproduced. Our main result is that the phase-space density power law, the Sersic density form and the linear $\beta - \gamma$ dependence constitute a consistent set of relations which obey the spherical Jeans equation and as such provide the framework for the dynamical modeling of DM halos.

Subject headings: cosmology: dark matter — galaxies: evolution — galaxies: formation — galaxies: halos — galaxies: kinematics and dynamics — galaxies: clusters

1. INTRODUCTION

In the standard cosmological model of structure formation the luminous matter is embedded in extended dark matter (DM) halos. The large scale structure emerges out of the primordial perturbation field *via* gravitational instability. The model further assumes the DM to be made of weakly interacting particles and its dynamics to be collisionless and therefore dissipationless (Padmanabhan 1993 for a review). The problem of the dynamics of DM halos can be formulated as the classical gravitational N -body problem, subject to the assumption of cosmological initial and boundary conditions. As such the problem can be very easily formulated, yet it defies any rigorous analytical treatment. The spherical top-hat model provides the main analytical tool for shading light on the problem, but its scope of validity is rather limited (Gunn & Gott 1972). The model can be extended to accommodate shell crossing (Gunn 1977) and cosmological initial conditions (Hoffman & Shaham 1985) resulting in the secondary infall model.

In the absence of a rigorous analytical theory the study of the evolution DM halos relies heavily on numerical N -body simulations. The advent of CPU power and improved numerical algorithm have led to a general consensus about the basic properties of DM halo, such as the spherically-averaged density profile, the spin and shape of halos. One of the pillars of the phenomenology of DM halos is the so-called NFW density profile (Navarro et al., 1997),

$$\rho_{\text{NFW}}(r) = \frac{4\rho_s R_s^3}{r(R_s + r)^2}. \quad (1)$$

It has been argued the NFW two-parameters fitting for-

mula provides a good approximation for DM halos found in a wide range of mass scales and cosmological models, and hence can be considered universal. Subsequent numerical simulations have basically confirmed the functional form of the NFW profile but some controversy concerning the asymptotic slopes of the profile at small and large radii persists (*e.g.*, Moore et al. 1998, Klypin et al. 2001, Jing & Suto 2000). Here we refer to these generalizations of the NFW fitting formula as the gNFW profile.

A second basic phenomenological finding is the so-called phase-space density (PSD) profile (Taylor & Navarro 2001). These authors defined the PSD profile by

$$Q(r) = \frac{\rho}{\sigma_r^3}, \quad (2)$$

where σ_r is the radial velocity dispersion. The basic finding of Taylor & Navarro (2001) is that the PSD profile follows a power law of the form

$$Q(r) \propto r^{-\alpha}. \quad (3)$$

Taylor & Navarro (2001) found $\alpha \approx 1.875$, and more recent studies find $\alpha = 1.92 \pm 0.01$ (Dehnen & McLaughlin 2005) and 1.94 ± 0.01 (Hoffman et al. 2007). Again, the power law behavior of $Q(r)$ is found over a wide range of mass scales and different cosmological models, suggesting a universal property of DM halos.

It has been recently suggested that a universal density slope-velocity anisotropy relation exists for relaxed DM halos (Hansen & Moore 2006). These authors found a linear relation between the velocity anisotropy parameter β and the density slope γ (< 0),

$$\beta = -0.2(\gamma + 0.8), \quad (4)$$

where

$$\beta(r) = 1 - \frac{\sigma_\theta^2 + \sigma_\phi^2}{2\sigma_r^2} \quad (5)$$

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and

$$\gamma = \frac{r}{\rho} \frac{d\rho}{dr}, \quad (6)$$

where σ_θ and σ_ϕ are the dispersions of the two transversal velocity components. Again, the linear relation has been found for a variety of halos obtained in various simulations and setups, suggesting a universal nature of the linear relation.

The phenomenological gNFW density profile, the PSD power law and the linear $\beta - \gamma$ relation are supposed to provide us clues about DM halos. The universal nature of these relations suggests that they hold over a broad range of scales and models, at least within the parameter space of the Cold Dark Matter (CDM)-like cosmogonies. The equilibrium structure of collisionless self-gravitating systems obey the Jeans equation (*e.g.*, Binney & Tremaine 1987), which relates the velocity second moments and the density field. Under the assumption of a spherical symmetry, the Jeans equation relates $\rho(r)$, $\sigma_r(r)$ and $\beta(r)$, and can be rewritten so as to relate $\rho(r)$, $Q(r)$ and $\beta(r)$. Lacking a fundamental theory that can predict even one of the above relations we are motivated to study the internal consistency of the three conditions and to find out whether one of these can be found to depend on the other two. Given the large scatter around the linear $\beta - \gamma$ relation (Hansen & Moore 2006, Hansen & Stadel 2006) we suspect that the $\beta - \gamma$ relation is the 'weakest' and less certain among the three relations. We therefore assume that the density of spherical halos follows an exact gNFW profile and that the PSD profile is a power law and solve the Jeans equation to obtain the β profile. This is to be compared with the linear $\beta - \gamma$ relation.

It has been recently suggested that the so-called Seric, also known as the Einasto (1965), profile provides a better fit to the density profile (Merritt et al. 2005; Prada et al. 2006; Gao et al. 2007). Given the very different functional form of the Einasto profile it will be considered here as an alternative to the NFW-like family of profiles and the associated $\beta(r)$ will be calculated. This will enable us to check the sensitivity of $\beta(r)$ to the assumed density fitting formula.

The structure of the paper is as follows. The Jeans equation is solved in §2 and the solutions of the β profile are given in §3 and discussed in §4.

2. THE JEANS EQUATION

The following model of DM halo is assumed here: the halos are in dynamical equilibrium; it is spherically symmetric; the halo density profile is given by the gNFW profile; and the PSD profile follows a power law. Such a halo should obey the Jeans equation (*e.g.*, Binney & Tremaine 1987),

$$\frac{d\sigma_r^2 \rho}{dr} + \frac{2\beta(r)}{r} \sigma_r^2 \rho = -\rho(r) \frac{GM(r)}{r^2}, \quad (7)$$

where $M(r)$ is the total mass enclosed within a spherical shell of radius r .

The PSD profile is expressed here in terms of the NFW scale radius, R_s (Eq. 12 below), the density, ρ_s , and the radial velocity dispersion, $\sigma_{r,s}$, evaluated at R_s . Namely,

$$Q(r) = \frac{\rho_s}{\sigma_{r,s}^3} \left(\frac{r}{R_s} \right)^{-\alpha}. \quad (8)$$

The $(r/R_s)^{-\alpha}$ scaling is found to persist throughout the entire evolution of individual DM halos along the main branch of their merger tree (Hoffman et al. 2007).

Rescaling the radial coordinate to $x = r/R_s$ the Jeans equation is rewritten as:

$$\left(\frac{\sigma_{r,s}^2}{R_s \rho_s^{2/3}} \right) \left[\frac{d}{dx} \left(\rho^{5/3} x^{2\alpha/3} \right) + \frac{2\beta(x)}{x} \rho^{5/3} x^{2\alpha/3} \right] = -\rho(x) \frac{GM(x)}{(R_s x)^2}. \quad (9)$$

Isolating $\beta(x)$ yields:

$$\beta(r) = -\frac{1}{2} \left[\frac{5x\rho'}{3\rho} + \frac{2\alpha}{3} + \frac{\rho_s^{2/3}}{\sigma_{r,s}^2 x^{2\alpha/3} \rho^{2/3}} \frac{GM(x)}{R_s x} \right] \quad (10)$$

Here the prime denotes a derivative with respect to x .

The density is assumed to follow a gNFW profile, using the functional form proposed by Zhao (1996), namely

$$\rho(r) = \rho_s \rho_{\text{gNFW}} \left(\frac{r}{R_s} \right), \quad (11)$$

and

$$\rho_{\text{gNFW}}(x) = \frac{2^{\epsilon-\mu}}{x^\mu (1+x)^{\epsilon-\mu}}. \quad (12)$$

The NFW profile corresponds to $\mu = 1$ and $\epsilon = 3$.

Given the PSD power law and the the gNFW profile the structure of a DM halo is determined by three parameters, *i.e.*, R_s , ρ_s and $\sigma_{r,s}$. In particular the velocity anisotropy profile is given by:

$$\begin{aligned} \beta(x) = & -\frac{\alpha}{3} + \frac{5(\mu + \epsilon x)}{6(1+x)} + \\ & \eta \frac{\pi 2^{1+(\epsilon-\mu)/3} x^{2-\mu/3} (1+x)^{2(\epsilon-\mu)/3}}{(\mu-3) x^{2\alpha/3}} \times \\ & {}_2F_1(\epsilon-\mu, 3-\mu, 4-\mu, -x), \end{aligned} \quad (13)$$

where ${}_2F_1(a, b, c, x)$ is the Gauss Hypergeometric function and $\eta = G\rho_s R_s^2 / \sigma_{r,s}^2$ is the dimensionless constant that determines β . Note that η is the square of the crossing-to-dynamical time ratio within R_s , hence it scales *inversely* with the (virial) ratio of kinetic-to-gravitational energy, T/W , within this radius. The kinetic energy term, T , is calculated here taking into account only the radial dispersion velocities, σ_r^2 .

The NFW density profile yields a β profile of the form

$$\begin{aligned} \beta_{\text{NFW}}(x) = & -\frac{\alpha}{3} + \frac{5(1+3x)}{6(1+x)} - \\ & \eta \frac{2^{5/3} \pi \left[\ln(1+x) - \frac{1}{1+1/x} \right] (1+x)^{4/3}}{x^{(2\alpha+1)/3}} \end{aligned} \quad (14)$$

Hence $\beta(x)$ has a minimum around $x \sim 0.1$ and a maximum somewhere between $x \sim 1$ and 10, depending on the value of η .

The Sersic density profile is given by

$$\rho(r) = \rho_s \rho_{\text{Ser}}(r/R_s) \quad (15)$$

where

$$\rho_{\text{Ser}}(x) = \exp \left[-2n(x^{1/n} - 1) \right], \quad (16)$$

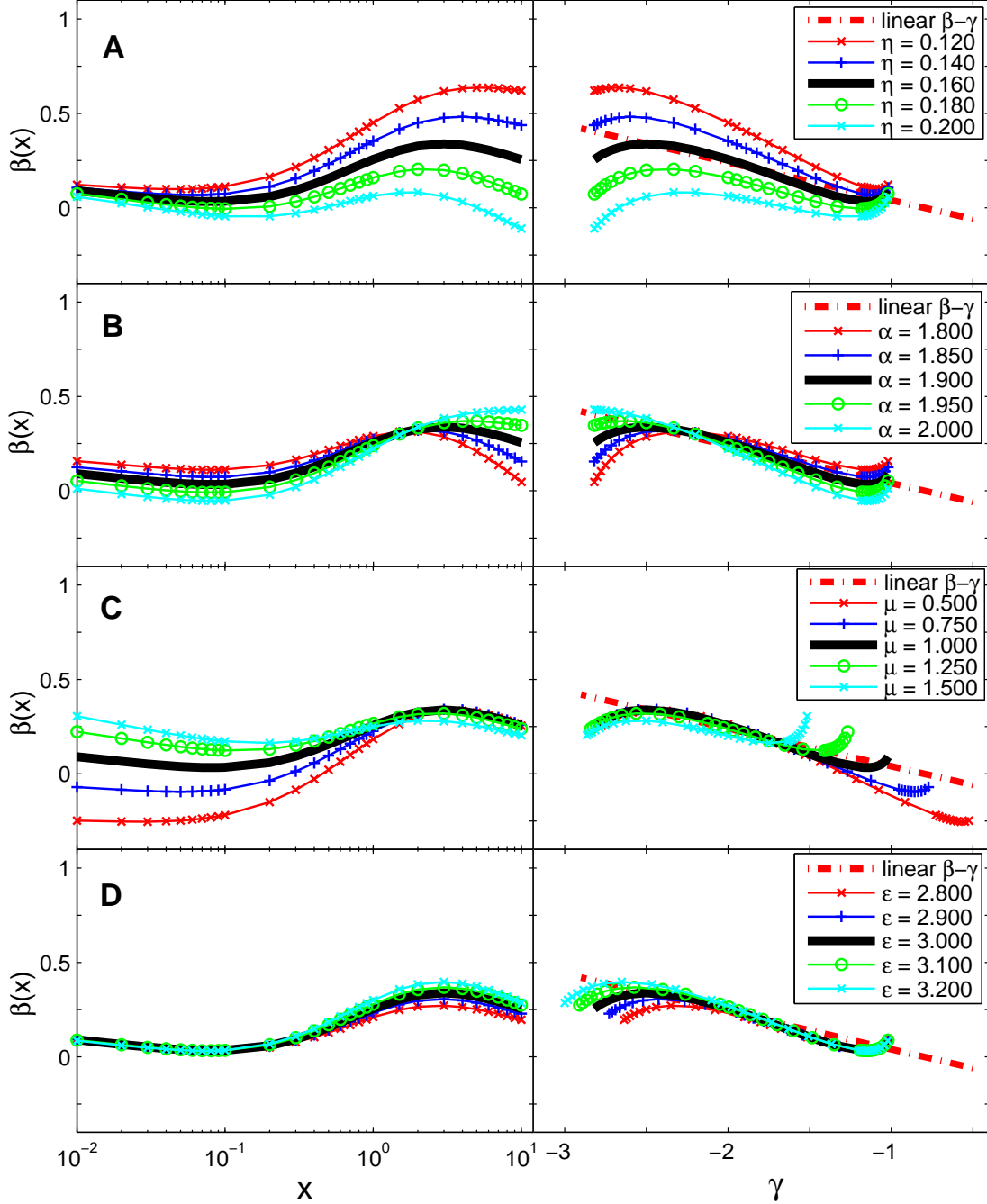


FIG. 1.— The dependence of the velocity anisotropy profile $\beta(x \equiv r/R_s)$ on the phase space density power law slope α , the inverse virial parameter $\eta = G\rho_s R_s^2/\sigma_{r,s}^2$, and the parameters of the generalized NFW density profile is presented. The left hand column displays the $\beta(x)$ and the right hand one the dependence of β on the logarithmic derivative of the density profile γ . The Hansen & Moore (2006) empirical fit to $\beta - \gamma$ is plotted in the right frames. A nominal model is assumed ($\eta = 0.16$, $\mu = 1.0$, $\alpha = 1.9$ and $\epsilon = 3.0$) and the parameters are varied around that model.

and n is the Sersic index (Prada et al. 2006). The Sersic density profile yields a β profile of the form

$$\beta_{\text{Ser}}(x) = \frac{1}{3 \exp^{2/3} [-2n(x^{1/n} - 1)]} \left\{ 8^{-n} x^{-2\alpha/3} (nx^{1/n})^{-3n} \right. \\ \left. \left(8^n \exp^{2/3} [-2n(x^{1/n} - 1)] x^{2\alpha/3} (nx^{1/n})^{3n} \right. \right. \\ \left. \left. (5x^{1/n} - \alpha) - 2 \exp[2n] \pi x^2 \eta \Gamma(1 + 3n) \right) \right\}, \quad (17)$$

$$+ 6 \exp[2n] \pi x^2 \eta \Gamma(3n, 2nx^{1/2}) \Big\}, \quad (17)$$

where $\Gamma(x)$ and $\Gamma(a, x)$ are the usual Γ and the incomplete Γ functions.

3. RESULTS

3.1. Generalized NFW Profile

Under the assumption of a gNFW density profile and a power law PSD profile the β profile is given by Eq. 13.

Here we choose a base model and change the four halo parameters, one at the time, to check the dependence of β (Fig. 1). The base model is an NFW density profile (*i.e.*, $\epsilon = 3$ and a $\mu = 1$ cusp), $\alpha = 1.9$ (Dehnen & McLaughlin 2005, Hoffman et al. 2007). Romano-Díaz et al. (2007) found that T/W within R_s is roughly 2, or somewhat larger. Therefore,

$$2 \approx \frac{-2T(< R_s)}{W(< R_s)} \approx \frac{3}{4\pi\eta}. \quad (18)$$

For the base model we choose $\eta = 0.15$. Fig. 1 shows β as a function of $x = r/R_s$ (left column) and γ (right column). The empirical $\beta = -0.2(\gamma + 0.8)$ relation (Hansen & Moore 2006) is plotted for reference.

Over the range of parameters studied here $\beta(x)$ has local minimum around $\sim 0.1R_s$ and a local maximum at a few $\times R_s$. At the minimum the velocity dispersion is nearly isotropic (*i.e.*, $\beta \approx 0$). Moving outwards to larger radii the velocity dispersion becomes more radial, as β increases. However, beyond a few R_s β starts to decrease and the dispersion becomes more isotropic. It is interesting to note that on very small scales, below $0.1R_s$, β monotonically decreases with r . Its value at $r = 0$ depends mostly on the slope of the cusp, μ . We find for $\mu = 0.5, 1.0$ and 1.5 the asymptotic values of β for $r \rightarrow 0$ are $-0.21, 0.2$ and 0.62 , respectively. (Note that for $\mu = 0.5$ β reaches a minimum at $r \approx 0.02R_s$ with $\beta \approx -0.25$.) The small scale behavior shows that NFW-like halos, obeying the phase-space density power law, can have steeper inner cusps by making their velocity dispersion more radial at the center. In other words, more radial orbits lead to a steeper inner density cusp.

Eq. 13, and consequently Fig. 1, displays the dependence of β on η — namely, an increase in η leads to a decrease in β . The latter can be understood as following. Increase in η means a decrease in T/W ratio, or equivalently an increase in $(v_c/\sigma_r)^2$, where v_c is the circular velocity at radius r . Here two possibilities exist: (1) if v_c^2 has increased at fixed σ_r^2 , the system responds by increasing the tangential dispersion velocities in order to remain in virial equilibrium and to be supported against the collapse. Note that while the overall angular momentum, J , of the system described by Eq. 7 is zero, the individual orbits have non-zero J and are randomly oriented which is in fact the source of the tangential velocity dispersion. An increase in v_c^2 then is translated into the increase in the tangential velocity dispersions in Eq. 10 and the associated decrease in β . (2) Alternatively, the growth in η can come from the decrease in σ_r^2 , while v_c^2 is fixed. The latter one requires that the tangential velocity dispersions stay unchanged, and, consequently, β will decrease by the same token.

3.2. The Sersic Profile

The $\beta(r)$ profile of the Sersic fit (Eq. 17) is presented in Fig. 2. The base model is taken to be $\eta = 0.16$, $\alpha = 1.90$ and a Sersic index of $n = 6.0$. The most striking feature of the β profile in the Sersic case is its much simpler functional form compared with the gNFW case. For the nominal $\alpha = 1.9$ case the $\beta - \gamma$ relation is very close to linear and β grows monotonically with x over the range of $10^{-2} < x < 10$. Moreover, the Sersic index of $n \approx 6$ very closely follows the Hansen & Moore (2006) linear relation. This is a surprising result. Both the gNFW and

the Sersic profiles have been invoked as fitting parametric models of the density profile in the DM halos, and as such they do not differ substantially. Yet, their resulting β profiles show considerable qualitative differences.

To gain further insight into the difference between the gNFW and the Sersic case we compare their corresponding density (Fig. 3) and β (Fig. 4) profiles. The Sersic profile has been proved to be a good fit for the density profile of DM halos over the range of $10^{-2} < r/R_{\text{vir}} \lesssim (1 - 2)$, where R_{vir} is the virial radius (Prada et al. 2006; Gao et al. 2007). The NFW fitting has been done over a comparable range. Given that the Sersic and the gNFW profiles are just fitting formulae and are not derived from some physical models, one should be cautious in using them beyond the range over which the fit has been performed. To prove that point, Figs. 3 and 4 have been plotted over the range of $10^{-2} < x < 10^2$. The Sersic profile shows an unphysical drop at $x \approx 20$ where β goes to minus infinity, namely the radial dispersion velocities vanish. This behavior is shared by all the Sersic models considered here, of $5 \leq n \leq 8$. However, DM halos do not extend that far and this behavior should not affect the modeling of DM halos. The anisotropy parameter does not converge to a certain limit on very small scales, $x \ll 10^{-3}$ say, and the monotonic dependence on the radius is not guaranteed.

3.3. Phase Space Density of the Total Velocity Dispersion

Given the radial PSD and the calculated β profile the PSD profile of the total velocity dispersion is easily evaluated. The total PSD is given by

$$Q_{\text{tot}}(r) = \frac{\rho}{\sigma_{\text{tot}}^3}, \quad (19)$$

where σ_{tot} is the dispersion of the full three dimensional velocity field. This is related to the radial PSD $Q(r)$ by

$$Q_{\text{tot}}(r) = Q(r)[3 - 2\beta(r)]^{-3/2}. \quad (20)$$

Assuming the nominal parameters of $\eta = 0.16$ and $\alpha = 1.90$ the total PSD profile is calculated for the NFW and the Sersic ($n = 6.0$) profiles (Fig. 5). In the two cases $Q_{\text{tot}}(r)$ is very well fitted by an $r^{-\alpha_{\text{tot}}}$ power law with $\alpha_{\text{tot}} = 1.847$ and 1.837 for the Sersic and NFW cases, respectively. The difference in the slope is insignificant and we conclude that $\alpha_{\text{tot}} = 1.84$. Yet, the fractional residual of the actual $Q_{\text{tot}}(r)$ from the fitted power law are as large as twenty percents for the NFW case but they are of the order of one percent in the Sersic case. The $\alpha_{\text{tot}} = 1.84$ was found before by Dehnen & McLaughlin (2005) and is in close agreement with the 1.82 of Faltenbacher et al. (2007).

4. DISCUSSION

The present work has been primarily motivated by the quest for understanding the linear $\beta - \gamma$ dependence (Hansen & Moore 2006) and its relationship to the two so-called universal relations, the PSD power law (Taylor & Navarro 2001) and the gNFW and the Sersic density profiles, that characterize the structure of DM halos. Here we have focused on whether the linear $\beta - \gamma$ relation is implied by the other two relations, and is it consistent with them.

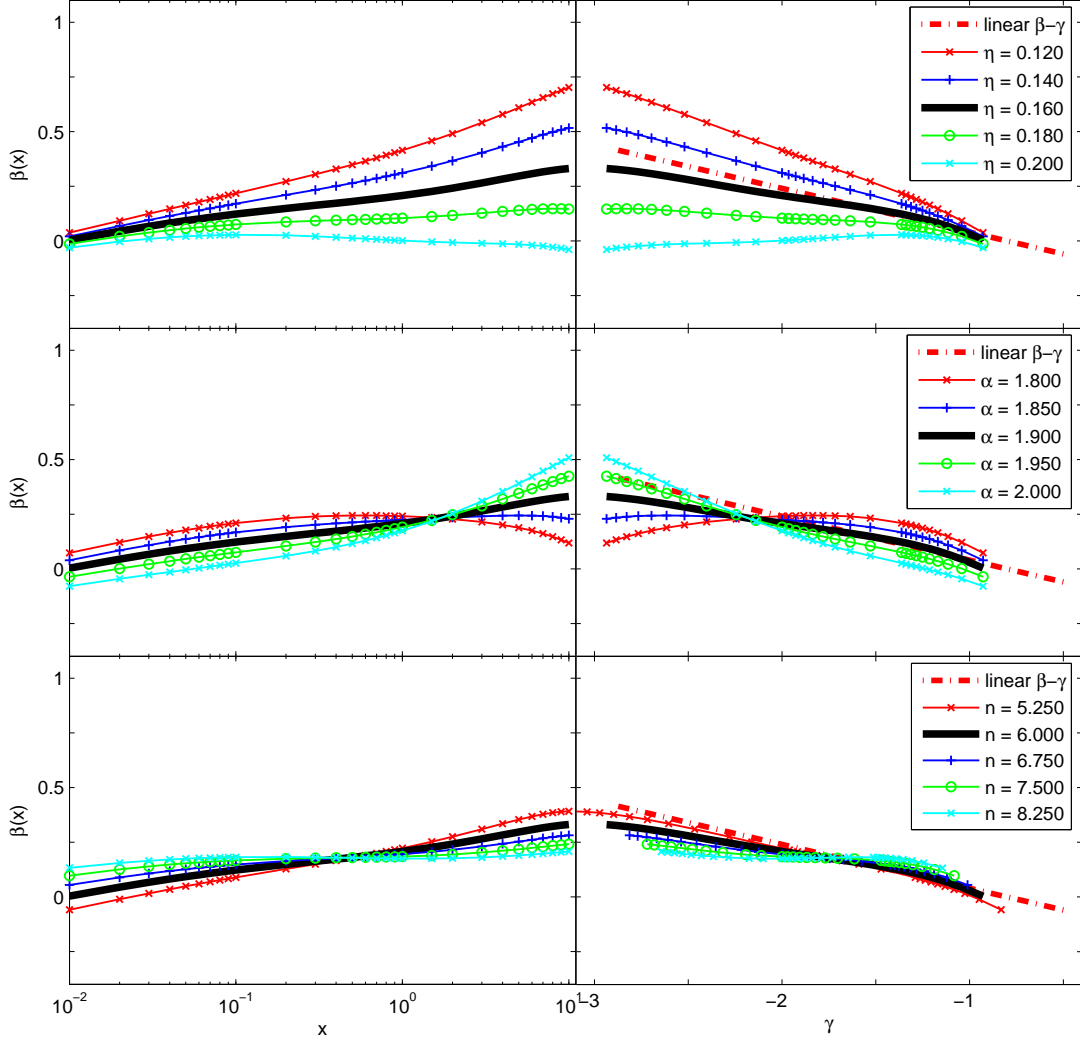


FIG. 2.— The dependence of the $\beta(r)$ profile on α , η and the Sersic index n is examined. The structure of the figure is identical to Fig. 1. The base Sersic model is taken here to have $1/n = 0.16$ and the lower panel shows the variation of the Sersic index.

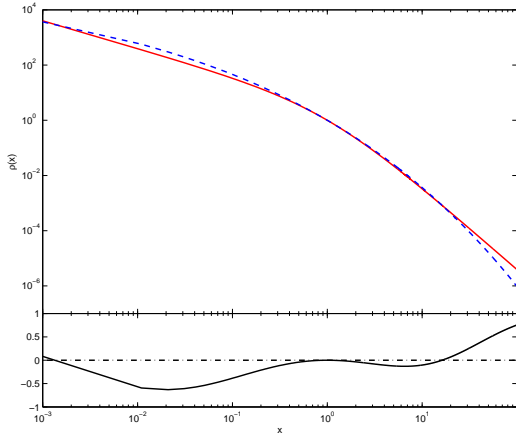


FIG. 3.— The NFW (red) and Sersic ($n = 6$, blue) density profiles. The bottom panel shows the fractional difference between the two models.

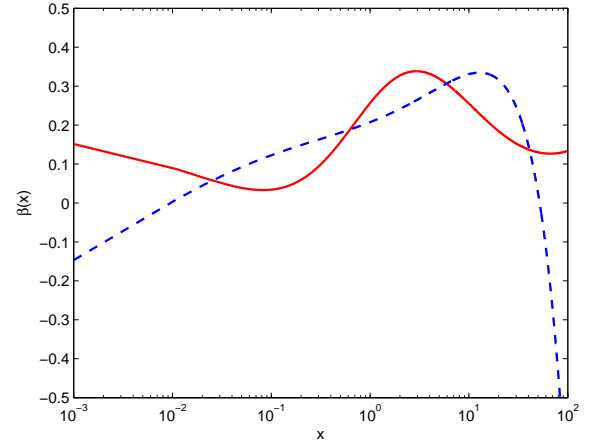


FIG. 4.— The NFW (red) and Sersic ($n = 6$, blue) β profiles. In both cases $\eta = 0.16$ and $\alpha = 1.9$ have been assumed.

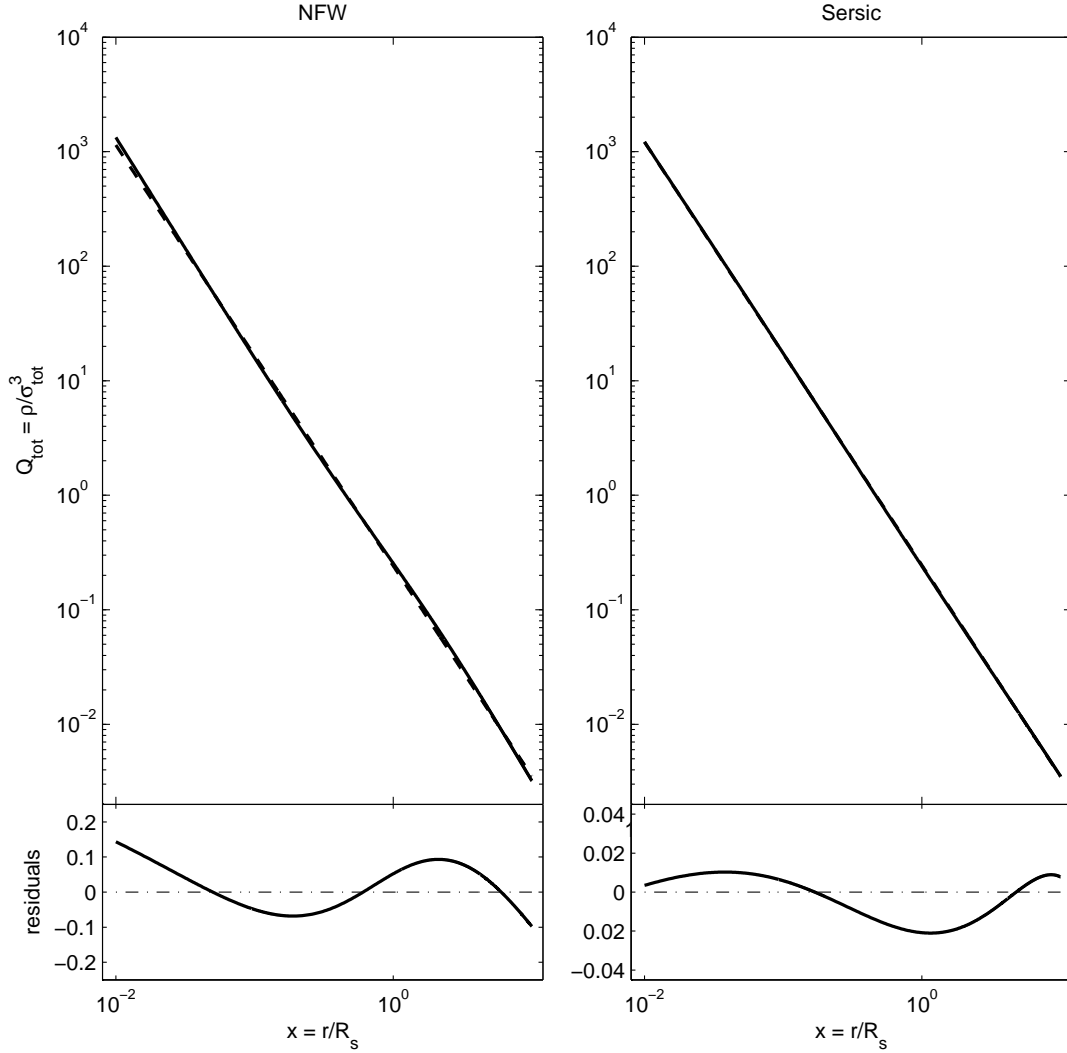


FIG. 5.— Given the power law $Q(r)$ profile and the calculated $\beta(r)$ the full velocity dispersion phase-space density profile, $Q_{\text{tot}} = \rho/\sigma_{\text{tot}}^2$, is easily evaluated. Here, Q_{tot} is evaluated for the NFW density profile, ($\eta = 0.16$, left panel) and the Sersic model ($n = 6$, right panel), assuming $\alpha = 1.9$ for both cases. The total velocity PSD closely follows a power law, $Q_{\text{tot}} \propto r^{-\alpha_{\text{tot}}}$, where $\alpha_{\text{tot}} = 1.84$. The fractional residual from a power law is presented in the bottom panels. Note that in the NFW the amplitude of the residual is about 10 percents and only about 1 percent in the Sersic case.

We find that for no choice of parameters an NFW-like density profile can yield a velocity anisotropy, $\beta(r)$, that is even in a rough agreement with the linear $\beta - \gamma$. This stands in a sharp contrast with the Sersic fitting formula for the DM halos density profile. For a Sersic profile index of $n \approx 6.0$ one recovers quite faithfully the linear $\beta - \gamma$ relation. For that model $\beta(r)$ is a monotonically increasing function of the radius. For the nominal model we find $\beta(x = 0.1) = 0.12$ and $\beta(x = 10) = 0.33$. A by-product of the solution of the Jeans equation is that the PSD of the total velocity dispersion follows a power law, namely $Q_{\text{tot}}(r) \propto r^{-\alpha_{\text{tot}}}$, with $\alpha_{\text{tot}} = 1.84$, for both the gNFW and the Sersic density profiles. Yet, in the fractional deviation in NFW-like case is of the order of twenty percents and in the Sersic case the deviation is of the order of one percent.

It has been realized in recent years that the Sersic profile provides a better fit to the density profile of DM

halos than the NFW model (Merritt et al. 2005, 2006; Prada et al. 2006; Gao et al. 2007). The present work substantiates and strengthens that fact and it strongly suggests that the Sersic model should be used for the dynamical modeling of DM halos.

The main result of the papers is that the three pillars of the DM halos phenomenology, namely the PSD power law, the Sersic density profile and the linear $\beta - \gamma$ relation, constitute a consistent set of relations that obey the Jeans equation. These relations provide a theoretical framework for a consistent dynamical modeling of DM halos.

A very different motivation for the calculation of the β profile has been to provide a practical tool for modeling the mass distribution in clusters of galaxies from kinematic data. A powerful way of modeling the clusters of galaxies is based on taking moments of the velocity distribution of clusters' galaxies and fitting them to the

solutions of the Jeans equation, under the assumptions of the NFW density profile and $\beta = \text{const.}$ (Lokas et al. 2006; Wojtak & Lokas 2007). The presently calculated β profile certainly provides a better approximation to the actual profile than the assumption of a constant value. We suggest that future analysis of the mass distribution of clusters will be based on the calculated profile of β .

We conclude the paper with a final note of caution. The analysis presented here applies strictly to spherical DM halos in virial equilibrium. Inspection of the formation of DM halos in cosmological simulations of CDM-like cosmologies reveals an ongoing process where halos grow continuously by a slow accretion and by major mergers, alternating between phases of dynamical equilibrium and violent off-equilibrium. Therefore, one expects the DM halos to show some deviations from a strict virial equilibrium and this might limit the validity of the solutions of the Jeans equation. DM halos are not isolated island structures in an otherwise unperturbed Friedmann uni-

verse. Halos are experiencing an ongoing smooth, and occasionally no so smooth, accretion and the boundary between the halo and outer universe is not easily defined. The stationary Jeans equation, on the other hand is applicable to isolated systems. It follows that even in stationary, seemingly relaxed systems the Jeans equation may not be strictly obeyed. A further complication arises from the deviation from sphericity of the DM halos. Cosmological simulations give rise to oblate and prolate ellipsoidal halos. This again might introduce a further deviation of the β profile of the DM halos from the solutions of the Jeans equation. These issues should be further investigated by means of N-body simulations.

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